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MEASUREMENT OF THE REFRACTIVE INDEX STRUCTURE COEFFICIENT C_n

by

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RDT&E Project No. 1T014501A31C

ABERDEEN PROVING GROUND, MARYLAND

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NJWright/RJSchutz/ilm
Aberdeen Proving Ground, Md.
December 1967

MEASUREMENT OF THE REFRACTIVE INDEX STRUCTURE COEFFICIENT C_n

ABSTRACT

The refractive index structure coefficient, C_n, is an important parameter for optical propagation in the atmosphere. A theoretical discussion of structure functions is presented to show the origin of the refractive index and temperature structure functions. The equipment used to measure temperature fluctuations in the atmosphere is discussed, and the calculation of the refractive index structure coefficient from these thermal measurements is shown. The details of the experimental evaluation of C_n and the results of C_n calculations are also included.

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INTRODUCTION

In a turbulent medium the velocity of the fluid flow at any point is defined by a vector which varies with the turbulence. The turbulence can be characterized by nine structure functions relating the velocity vectors at two points in the flow. A measure of the homogeneity of the medium can be expressed by the structure coefficients derived from these functions.

Using a theoretical model developed by V. I. Tatarski for atmospheric turbulence affecting optical propagation, a method of directly measuring the temperature fluctuations between two points normal to the wind velocity is employed to evaluate the refractive index structure coefficient C_n. When this meteorological determination of C_n is correlated with the observed characteristics of monochromatic light propagated through the atmosphere, the applicability of the body of theory developed by Tatarski and others to optical propagation in the atmosphere can be determined.

THEORETICAL BACKGROUND

The structure functions of a turbulent flow are of the form

$$D_{ij}(\vec{r}) = \overline{\left[V_i(\vec{r}_1) - V_i(\vec{r}_2)\right]\left[V_j(\vec{r}_1) - V_j(\vec{r}_2)\right]}, \quad \text{for } i,j = 1,2,3, \quad (1.1)$$

where the V_{ij}'s are the velocity components of the medium at the points $\vec{r}_1$ and $\vec{r}_2$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$. The bar over the right-hand side of Equation (1.1) indicates time averaging of the product beneath it. (The time average gives a measure of the expected value of the quantity.) Thus, the structure functions are related to the expected values of the covariance of the velocities at the two points.

If a condition of local isotropy is assumed to exist in the turbulent medium and the two points $\vec{r}_1$ and $\vec{r}_2$ are in a plane perpendicular to the flow and within the range of isotropy, the tensor D_{ij} can be expressed as a function of $r = |\vec{r}|$, i.e., the distance between the two points. The longitudinal structure function $D_{rr}(r)$ and the transverse structure function $D_{tt}(r)$ then determine $D_{ij}(\vec{r})$. Making the further assumption of homogeneity of the flow, one can express these two structure functions in terms of each other and thus characterize the turbulent flow by one structure function depending only on r. When the separation of the two points is large compared to the inner scale of the turbulence, denoted by l_o (the dimensions of the smallest turbulent eddies in the flow) but small compared with the outer scale, L_o (the largest dimension for which the medium can be assumed to be isotropic), the structure functions are directly proportional to $r^{2/3}$. The validity of this "two-thirds law," first obtained by Kolmogorov[1*] and Obukov,[2] has been investigated by other experimenters[3,4] and found to hold reasonably well. The square root of the coefficient of proportionality is called the structure coefficient of the flow, and the size of this coefficient is a measure of the local inhomogeneity of the medium. For a steady-state laminar flow, this coefficient would be zero as the medium would be totally homogeneous and the velocities at all points in the medium the same.

Treating temperature as a "conservative passive additive" of the atmosphere, i.e., a parameter that has no direct influence on atmospheric turbulence, Obukov[5] related the structure functions of turbulent flow to the structure function of the atmospheric temperature field. From this work Tatarski[6] derived an expression for the refractive index structure function:

$$D_n(r) = C_n^2 r^{2/3} , \qquad \text{for } l_o \ll r \ll L_o , \quad (1.2)$$

where C_n is the structure coefficient of atmospheric refractive index and may be thought of as a measure of the optical "strength" of atmospheric turbulence.

The analogous expression for the temperature structure function developed by Obukov is

$$D_T(r) = C_T^2 r^{2/3} , \qquad \text{for } l_o \ll r \ll L_o , \quad (1.3)$$

Superscript numbers denote references which may be found on page 19.

where C_T is the structure coefficient of temperature. Equations (1.2) and (1.3) give the theoretical relationships necessary to evaluate C_n from the measurement of atmospheric temperature fluctuations.

INSTRUMENTATION

The instrument used to measure the temperature fluctuations between two points in the atmosphere is essentially a Flow Corporation Hot-Wire Anemometer modified for use as a high-speed atmospheric thermometer. The Flow Corporation equipment consists of two high-speed temperature probes, the Anemometer Bridge Model HWB-3, a secondary voltage supply Model HWI-3, and a Random Signal Voltmeter Model 12A-1, Figure 1. Data are read with a digital voltmeter and recorded on paper tape by a digital clock/printer. The digital voltmeter samples the output signal of the continuous reading Random Signal Voltmeter at 1-second to 1-minute intervals. The clock/printer records the time and the voltage reading for each sample.

The temperature probes utilize fine tungsten wires of 10 microns in diameter and approximately 0.3 centimeter in length as sensing elements. For precision temperature measurements, an operating current of 1 ma is used in these wires. The resistance of the tungsten wires in the range of atmospheric temperatures is essentially a function linear with temperature and may be written

$$T = K_R R + T_R \ . \tag{2.1}$$

T is in degrees Kelvin, and R is in ohms. T_R is a constant equal to the temperature corresponding to zero resistance on the straight line curve. When taking the temperature difference measurement between the two probes, a difference in the resistance constant K_R for different probes introduces an error in the resistance measurement. The probes are balanced to give a zero temperature difference for any change in average air temperature in order to minimize this error. The difference in the two resistance constants is small, and therefore the error made in measuring the small

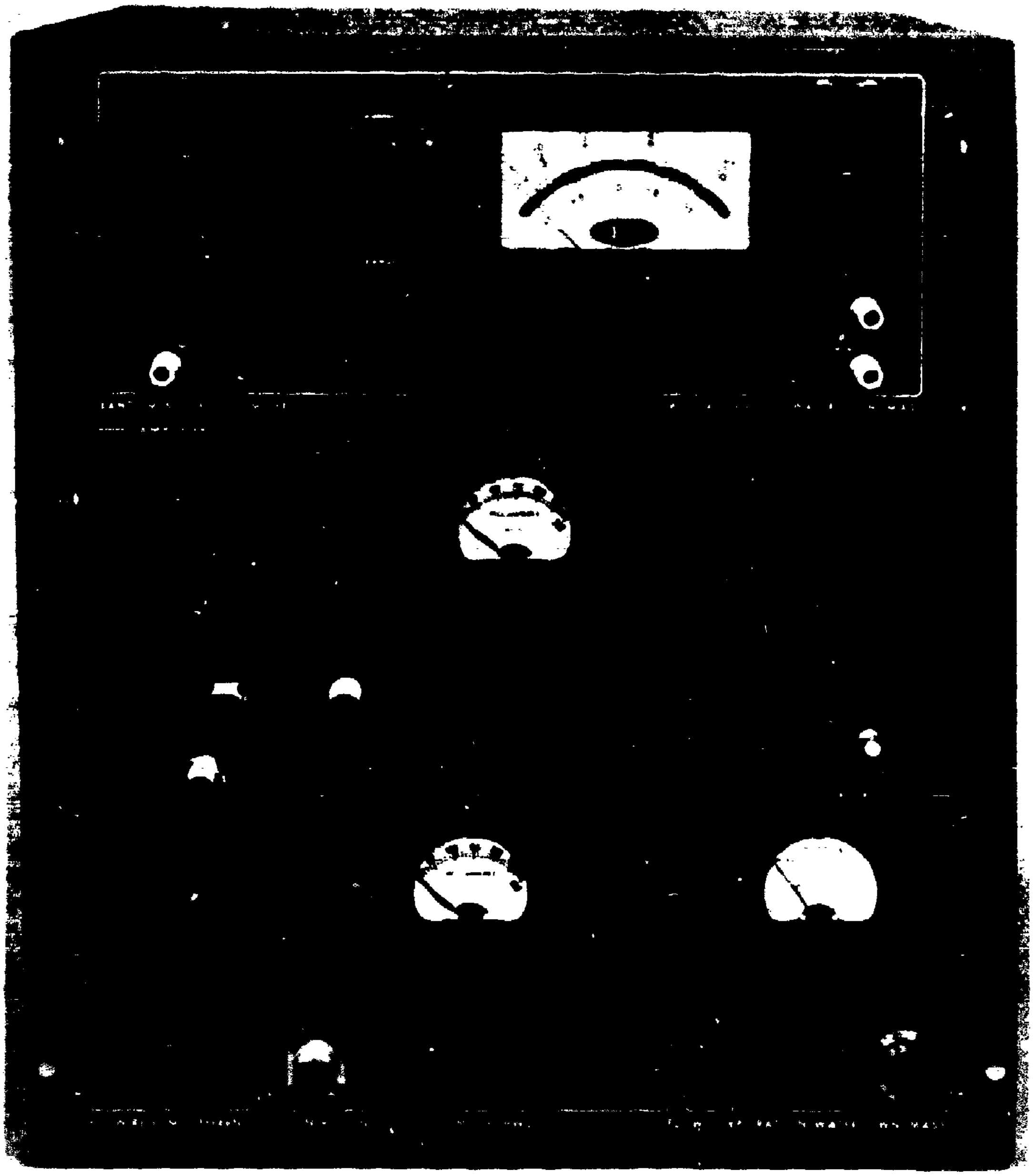

Figure 1. C_n measurement instrumentation
(Photograph courtesy of Flow Corporation)

short-time temperature differences relating to C_n is negligible. The average of the two constants is used for calculation proposes. The difference in temperature between the two probes can be written approximetely as the difference in their resistance multiplied by the average resistance constant $\bar{K}_R$,

$$\Delta T \simeq \bar{K}_R \Delta R \; . \tag{2.2}$$

With a current of 1 ma in the wires, resistance in ohms is directly equivalent to voltage measured in millivolts. Therefore,

$$\Delta T \simeq \bar{K}_R \Delta V \; . \tag{2.3}$$

The thin tungsten wires insure fast response of the probes to atmospheric temperature fluctuations, but the large surface area to volume ratio makes the probes sensitive to solar heating. It is assumed any solar heating is equal for both probes and so will have no effect on the temperature difference measurement.

The two probe voltages are balanced through the bridge circuits of the HWB-3 equipment using the galvanometer included in this unit until a null is achieved across the bridge. This calibration is made with the temperature probes enclosed in a reflecting wooden box to insure that both probes are exposed to the same average air temperature.

After the two probe voltages are nulled, the probes are securely mounted at a separation distance of 30 centimeters horizontally about the axis of a wind vane, Figure 2. The wind vane maintains the probes in a perpendicular orientation with respect to the wind flow. The 30-centimeter distance satisfies the conditions of Equations (1.2) and (1.3) as the inner scale of atmospheric turbulence is on the order of a few millimeters, while the outer scale affecting optical propagation is on the order of 2 meters, the beam height above the ground. When the two probes are exposed to the air, the fluctuations in atmospheric temperature produce a continuous

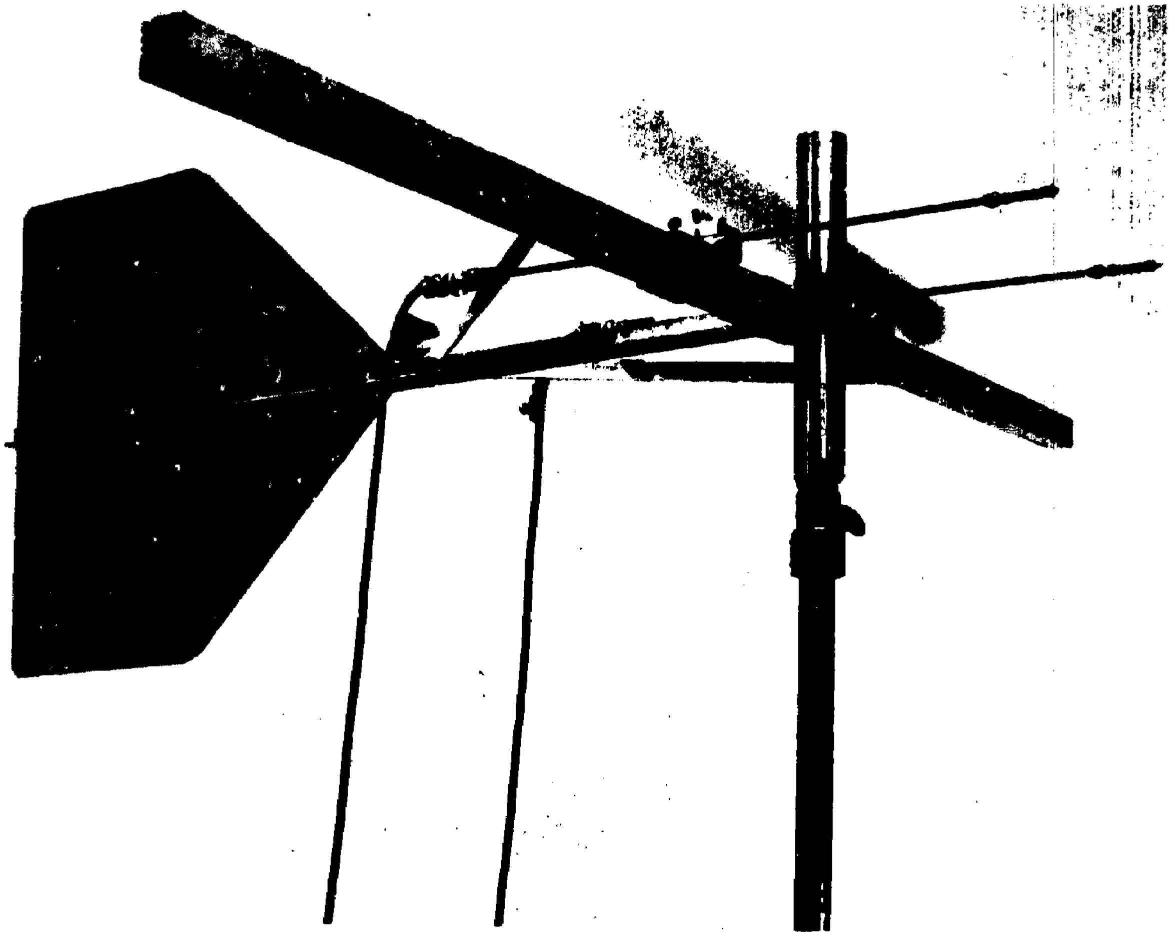

Figure 2. Temperature probes mounted on wind vane

voltage signal in the bridge circuit corresponding to the differences in temperature at the two probes as indicated in Equation (2.3). This signal is then amplified and fed into the 12A-1 Random Signal Voltmeter.

The Random Signal Voltmeter processes voltages by means of an internal thermal block with a time constant of approximately 16 seconds. The response of the thermal block varies as the square of the input voltages. Thus, the Random Signal Voltmeter reading is the 16-second time average of the square of the voltage differences and is directly proportional to the time averaged square of the temperature gradient at the two probes by the constant $\bar{K}_R^{\,2}$;

$$\overline{\Delta T^2} = \bar{K}_R^{\,2}\,\overline{\Delta V^2} \; . \qquad (2.4)$$

Taking the square root of Equation (2.4) to obtain the time average root mean square temperature gives

$$\Delta T_{RMS} = \bar{K}_R \Delta V_{RMS} \; . \qquad (2.5)$$

A potential source of error arises from the small signal to noise ratios encountered in the measurement. The use of internal battery voltage supplies for probe biasing eliminates the possibility of introducing interior random or cyclic noise. The remaining noise is due to random signal pickup in the cables leading from the probes to the bridge. As this noise level (read during calibration) is relatively constant, it can be subtracted from the final signal to greatly reduce this error. Appendix A gives the detailed calculations of the temperature difference including the correction for noise level.

DETERMINATION OF C_n

From the preceding discussion of the structure functions of a turbulent medium, we can define the structure functions of temperature and refractive index as

$$D_T(r) = \overline{\left[T(r_1) - T(r_2)\right]^2} \qquad (3.1)$$

$$D_n(r) = \overline{\left[n(r_1) - n(r_2)\right]^2} \qquad (3.2)$$

where $$r = |r_1 - r_2|$$

and the average bars indicate time averaging. Equation (3.1) is an expression for the time averaged square of the temperature difference between r_1 and r_2 and can be related to the readings of the Random Signal Voltmeter by Equation (2.4)

$$D_T(r) = \overline{\Delta T^2} = \bar{K}_R{}^2\overline{\Delta V^2} . \qquad (3.3)$$

Using Equations (1.2) and (1.3) presented earlier, we obtain

$$C_n{}^2 = C_T{}^2(D_n/D_T) . \qquad (3.4)$$

The quantity D_n/D_T in this expression is a function of temperature, but the dependence is as $1/T^4$, where T is in degrees Kelvin. For small temperature changes D_n/D_T can be considered a constant. Appendix B justifies this approximation and gives the method used to evaluate D_n/D_T.

Using Equations (1.3) and (3.3), Equation (3.4) can be written as

$$C_n{}^2 = \bar{K}_R{}^2(D_n/D_T)\ \overline{\Delta V^2}r^{-2/3} . \qquad (3.5)$$

Taking square roots and combining the two constants gives a concise expression for the structure coefficient

$$C_n = CV_{RMS}r^{-1/3} , \qquad (3.6)$$

where the constant C is equal to $\bar{K}_R(D_n/D_T)^{1/2}$.

The refractive index structure coefficient C_n was evaluated for 18 hours on 18 August 1967 from 0600 to 2200 hours. The results of the measurements are plotted against time of day in Figure 3. The Random Signal Voltmeter readings were averaged to 10-minute intervals for this analysis. The isolated points for the first few readings result from gaps in the data caused by repeated calibration checks at the start of the experiment. The maximum value of C_n is indicated at 1300 hours, ESDT, and therefore coincides with the sun's zenith.

Data were taken on 22 August 1967 from 0900 hours to 1400 hours. The results of the C_n calculations for this day are plotted in Figure 4 (5-minute average) and Figure 5 (10-minute average). The maximum value for C_n again occurs near 1300 hours.* Also important is the full range of C_n on both these days exhibiting variations of nearly 100 to 1.

It is paradoxical that a structure function describing an optical parameter is better quantified by a thermal rather than an optical measurement. However, it has been demonstrated that the index structure coefficient, C_n, cannot be correctly computed through the well-known equation given by Tatarski** for the amplitude variance in terms of the path length, wave number, and the index structure coefficient.[7] It is apparent that measurements of appropriate meteorological parameters are needed before an adequate understanding of optical propagation in the atmosphere can be gained.

**It is expected that C_n should reach a peak at this time as the solar flux to the earth's surface is at a maximum producing the greatest heat transfer at the air-ground interface.*

***See Reference 6, p. 188, Equation (9.43).*

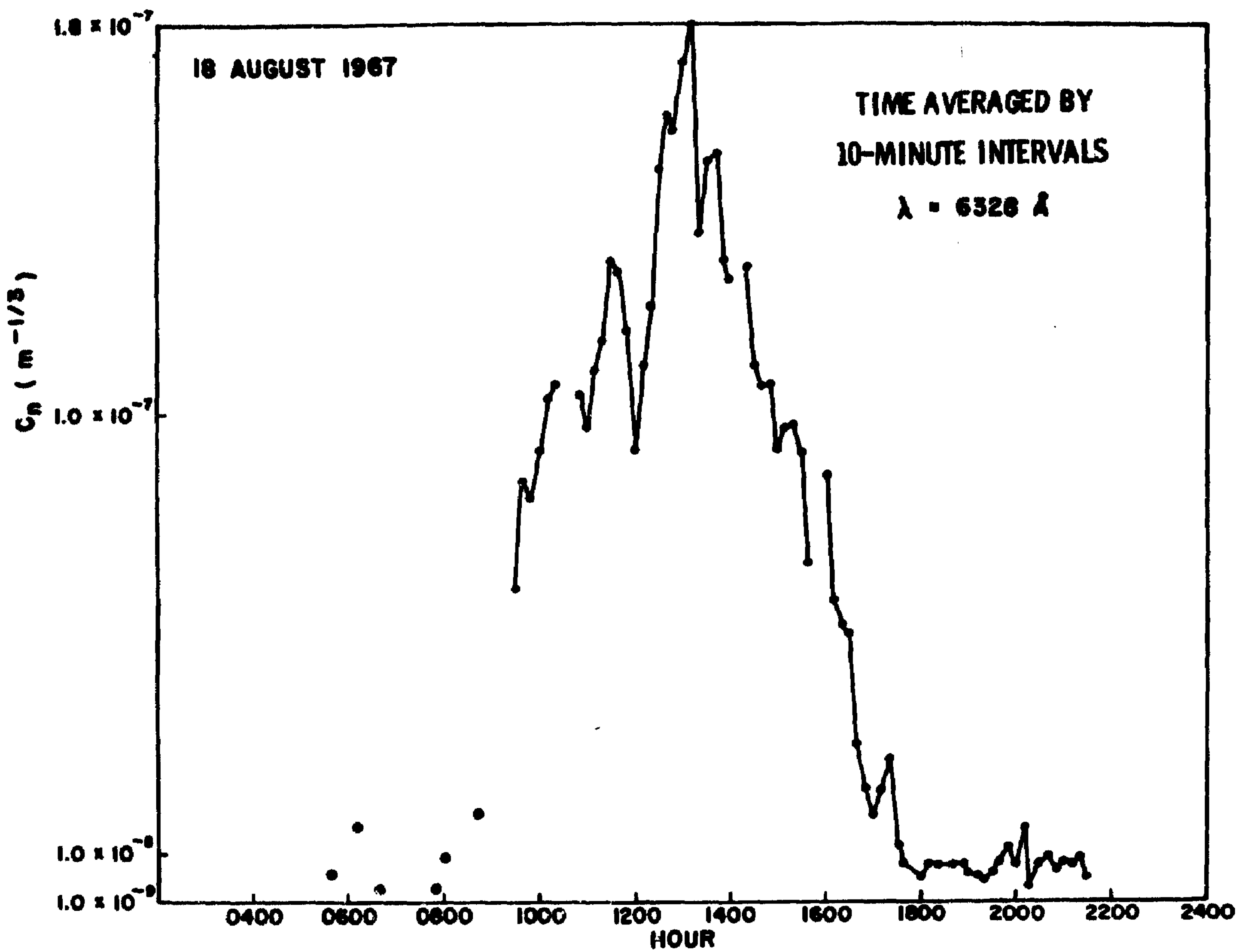

Figure 3. C_n versus hour of day (EDST), 18 August 1967, 10-minute average

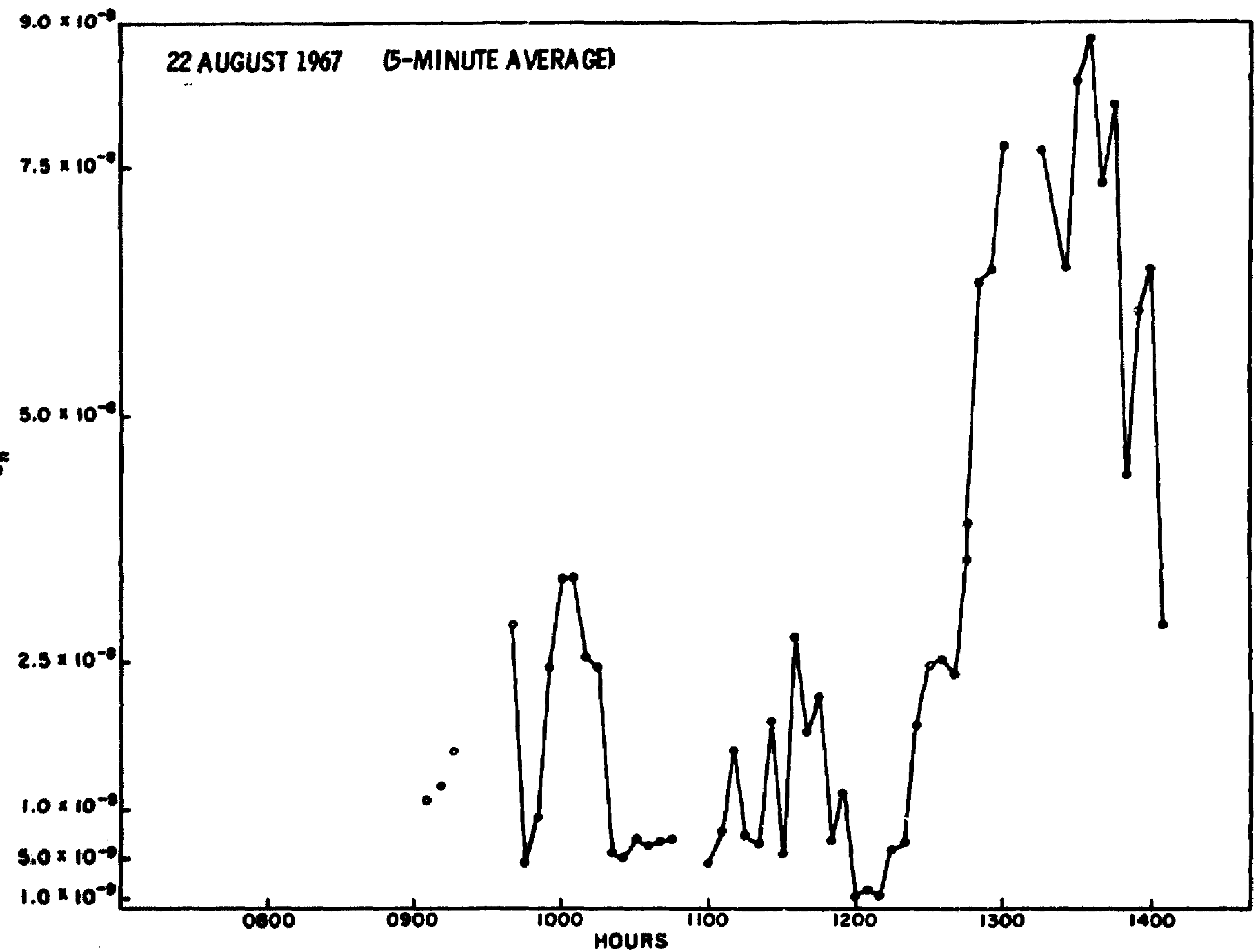

Figure 4. C_n versus hou of day (EDST), 22 August 1967, 5-minute average

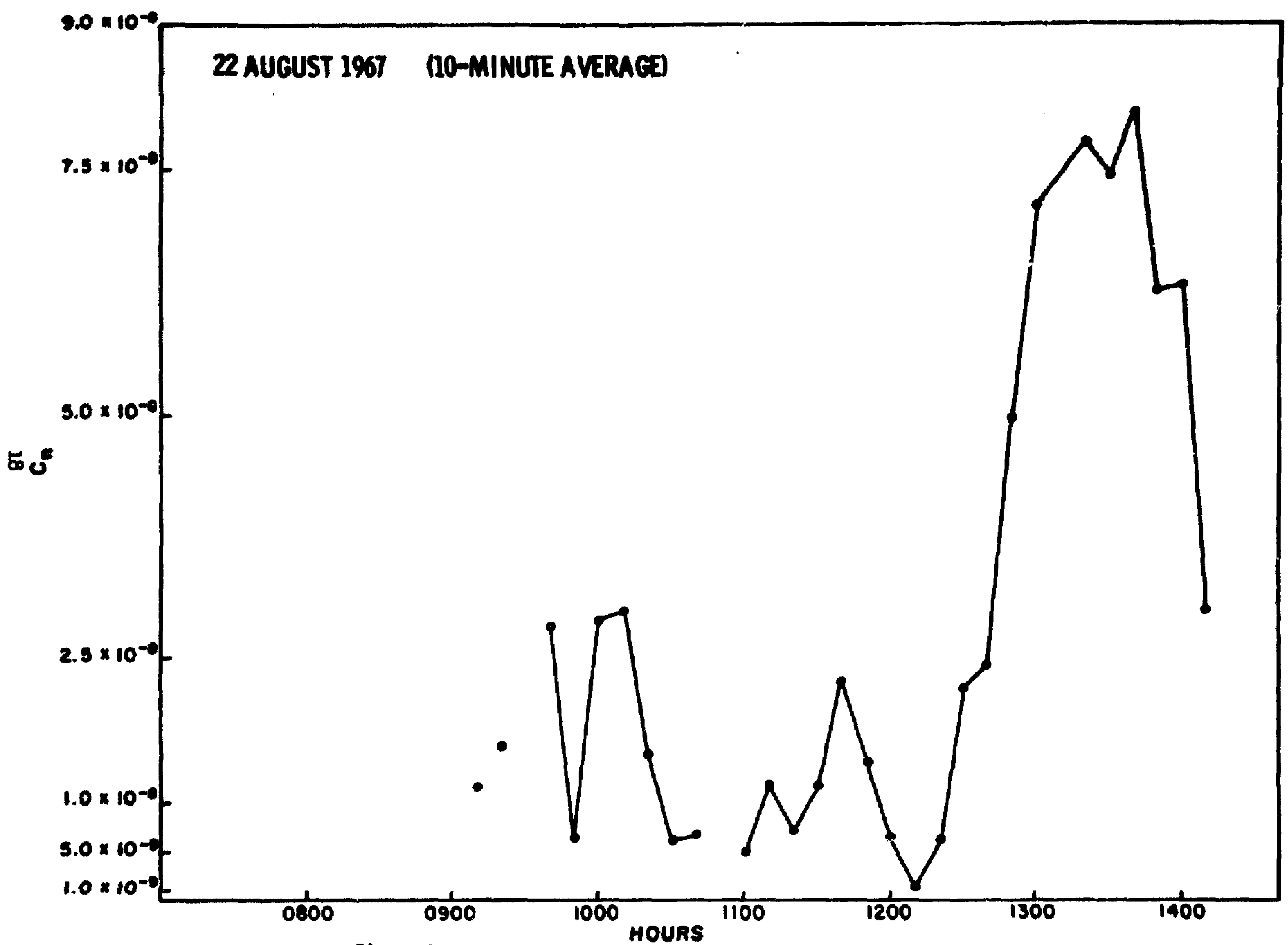

Figure 5. C_n versus hour of day (EDST), 22 August 1967, 10-minute average

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APPENDIX A

The voltage at the meter output of the Flow Corporation 12A1 is related to the reading on the RMS meter in the following way:

$$E_{mo} = K\left(\frac{E_{in}}{E_{fs}}\right)^2 , \tag{A-1}$$

where E_{mo} = voltage at meter output terminals
E_{in} = meter reading (RMS value of input voltage)
E_{fs} = full-scale voltage on the range in use
K = a meter constant, approximately 5 mV.

The thermal probes have the following linear relationship with temperature:

$$T = K_R R + T_R , \tag{A-2}$$

where T is the temperature (degrees Kelvin), R is the resistance (ohms), and K_R and T_R are constants that are found by calibrating the probes.

The temperature difference then is

$$\Delta T = K_R \Delta R . \tag{A-3}$$

If the probes are biased with a constant current of 1 ma,

$$\Delta T = K_R \Delta V , \tag{A-4}$$

where ΔV is measured in millivolts.

If both probes were to have the same K_R, then

$$\Delta T_{RMS} = K_R \Delta V_{RMS} , \tag{A-5}$$

where ΔV is the voltage (divided by the appropriate gain factor) displayed on the random signal voltmeter. Usually the two K_R's are nearly the same; therefore, averaging the slopes of the two probe curves, the approximation

$$\bar{K}_R = \frac{K_{R(1)} + K_{R(2)}}{2} \tag{A-6}$$

is used. Relating Equations (A-1) and (A-5) we now find

$$\Delta T_{RMS} = \frac{1}{50}\bar{K}_R\left(\frac{E_{mo}}{K}E_{fs}\right)^{1/2} . \tag{A-7}$$

The factor of 1/50 is included because we are assuming all measurements will be made with galvanometer range switch on the 0.2 mV scale.

Equation (A-7) would be correct if the noise level of the instrument were zero. A noise level is present, however, and must be subtracted from the readings to obtain an accurate temperature difference measurement. Accounting for the system noise gives finally,

$$\Delta T_{RMS} = \frac{1}{50\sqrt{K}}\bar{K}_R\left(E_{mo}E_{fs} - E_{mo_n}E_{fs_n}\right)^{1/2} , \tag{A-8}$$

where the "n" subscript denotes noise readings recorded during calibration of the equipment.

APPENDIX B

In order to show that the quantity D_n/D_T is proportional to $1/T^4$, where T is temperature in degrees Kelvin, the following relationship can be used giving the refractive index of air as a function of temperature, pressure, and wavelength:

$$(n - 1) = 77.6P/T(1 + 0.0075\lambda^2) \times 10^{-6} \tag{B-1}$$

for pressure P in millibars and wavelength λ in meters. For small temperature changes atmospheric pressure can be taken to be constant allowing the change in index of refraction with respect to temperature to be written as

$$\Delta n/\Delta T = C/T^2 \ , \tag{B-2}$$

where C is the value of the constant parameters. From Equations (3.1) and (3.2), assuming the time average to be over a short period in which temperature does not vary greatly, the time rate of change of the refractive index can be given as

$$D_n/D_T = \overline{(\Delta n)^2}/\overline{(\Delta T)^2} = C^2/T^4 \ . \tag{B-3}$$

Since the average temperature in degrees Kelvin is large with respect to the temperature differences measured, (on the order of tenths of degrees Kelvin) a constant value for D_n/D_T evaluated at the average air temperature can be substituted in Equation (3.5).

For the actual calculation of D_n/D_T a more precise formula for atmospheric index of refraction credited to Berrell and Sears was used.*
This equation includes corrections for vapor pressure in the atmosphere and higher order wavelength factors. In order to evaluate the difference quotient for the average temperature, the Barrell and Sears formula was evaluated for the average temperature plus and minus one degree. The average of the two differences with respect to average temperature was then used for D_n/D_T in the C_n calculations.

*<u>Smithsonian Meteorological Tables</u>, *6th ed., Washington, D.C., Smithsonian Institute, p. 389.*

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13. ABSTRACT

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Atmospheric Turbulence						
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